SEISMIC RESPONSE OF AGED CONCRETE DAM CONSIDERING INTERACTION OF DAM AND RESERVOIR IN COUPLED WAY

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ABSTRACT

The degradation of concrete due to various hygro-chemo-mechanical actions is inevitable for the structures particularly built to store water. Therefore, it is essential to determine the material properties of dam like structures due to ageing in order to predict the behavior of such structures after certain age. The degraded material properties are calculated by introducing isotropic degradation index. The predicted material properties are used to study the behavior of aged dam-reservoir coupled system. Both the dam and infinite reservoir are modeled by finite elements. Displacement and pressure are considered as nodal variable for dam and reservoir respectively. The effect dynamic interaction between dam and reservoir are calculated in a coupled manner. The parametric study reveals that the responses of dam-reservoir system are unexpectedly large at an age when system frequency matches with the exciting frequency. The outcomes of the present study indicate the importance of the consideration ageing effect of concrete exposed to water for the safe design of dam throughout its life time.

Keywords: Hygro-chemo-mechanical; pine flat dam; dam-reservoir system; isotropic degradation index; fluid-structure interaction; finite element method.

1. INTRODUCTION

A concrete gravity dam is a massive submerged structure, and is built across the river mainly to harness the potential energy of river water for generation of hydroelectricity. For the design of an earthquake-resistant dam and the seismic safety evaluation of existing dam, hydrodynamic pressure is recognized to be major disturbing force on dam. Distribution of hydrodynamic pressure on the vertical face of rigid dam was first established by Westergaard [1], based on 2-D hydrodynamic theory. Since then, several investigators have contributed to the subject. Chopra [2] developed an analytical solution of the wave equation

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to obtain the hydrodynamic pressure on the vertical face of the structures during earthquake. The hydrodynamic pressure distribution on the rigid structure having inclined upstream face of constant slope was presented analytically by Chwang [3]. All the analytical methods presented are suitable only for rigid structures. However, in most of the practical cases, the structures are elastic in nature and the estimated hydrodynamic will be more realistic if the interaction between an elastic structure and a compressible fluid has been incorporated properly in the analysis procedure. Some simplified analyses are available in which fluid-structures interaction is studied in a decoupled manner.In this type of analysis, the fluid response is first obtained assuming the structure as rigid and resulting pressure field is imposed on the structure to obtain the response of structure. But the process lead to the development of unsound design,particularly, for the case of resonance between structures and fluid. In recent years,some researchers [4-9] are using indirect iterative method to deal the fluid- structure interaction problems. In this method, the hydrodynamic pressure in fluid domain is first determined considering structure as rigid. The resulting pressure exerts forces on the adjacent structure. Due to this additional forces structure undergoes new displacement. The fluid domain is solved again with the calculated displacement to get the response of the elastic structures. The process is continued till a desired level of convergence in both pressures and displacements areachieved. The major advantage of this method is that the accuracy of this method depends on pre assign tolerance value and the computational time at each time step becomes larger particularly for the case when the structure has been considered as flexible.To compensate the inadequacies of this analyses procedure, an efficient direct finite element approach is used to study the fluid–structure interaction problems [10-13]. In this method fluid and structure are coupled and solved as one system.

To evaluate the dynamic behavior of the concrete gravity dam most of the researchers assumed that the modulus of elasticity of concrete of dam may remain constant throughout its design life. But, due to aging, the dams are subjected to severe environmentaleffects, which lead to degradation of the dam concrete.Since the dam face is in constant contact with water, concrete degradation due to hygro-mechanical loading is inevitable andshould be considered in the analysis procedure. Kuhl et al. [14] used chemo-mechanical model to obtain deterioration of cementitious materials. Gogoi and Maity [15] and Barman et al. [16] proposed different empirical equations based on experimental data to predict the modulus of elasticity at different ages of concrete.

In this paper,the behavior of concrete gravity dam at its different ages has been studied considering dam-reservoir interaction in direct coupled way.In finite element analysis, the pressure and displacement are considered as independent nodal variablesfor reservoir and damrespectively.The infinite reservoir is truncated at a certain distance with an effective truncation boundary condition. The elasticity properties of concrete at different ages are determined from the hygro-chemo-mechanical degradation model. A computer code in MATLAB environment has been developed to obtain seismic response of concrete dam-reservoir coupled system at its different ages.

2. THEORETICAL FORMULATION

2.1 Theoretical formulation for dam
The equation of motion of a dam like structure subjected to external forces can be written in standard finite element form as

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\}$$ (1)

where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrix of structure respectively, $\{\ddot{u}\}$, $\{\dot{u}\}$ and $\{u\}$ are nodal accelerations, velocities and displacements, $\{F\}$ is the nodal forces including hydrodynamic forces due to adjacent reservoir. In present investigation the structure has been discretized by two dimensional eight node rectangular elements. The dam body is assumed to be in a state of plane strain. The structural Rayleigh damping can be expressed as

$$[C]=a'[M]+b'[K]$$ (2)

where a' and b' are called the proportional damping constants. The relationship between a', b' and the fraction of critical damping at a frequency ω is given by the following equation.

$$\xi'=\frac{1}{2}\left(a'\omega+\frac{b'}{\omega}\right)$$ (3)

Damping constants a' and b' are determined by choosing the fraction of critical damping ξ_1' and ξ_2' at two different frequencies ω_1 & ω_2 and solving simultaneously equations a' and b'. Thus,

$$a'=\frac{2(\xi_2'\omega_2-\xi_1'\omega_1)}{(\omega_2^2-\omega_1^2)}$$
$$b'=\frac{2\omega_1\omega_2(\xi_2'\omega_1-\xi_1'\omega_2)}{(\omega_2^2-\omega_1^2)}$$ (4)

Usually, ω_1 is taken as the lowest natural frequency of the structure, and ω_2 is the highest frequency of interest in the loading or response. In the present study, the fraction of critical damping for both the frequencies are chosen as the same i.e. $\xi_1''=\xi_2''=\xi'$. Thus, above equation may be expressed as

$$a' = \frac{2\xi'}{(\omega_2 + \omega_1)}$$

$$b' = \frac{2\xi'\omega_1\omega_2}{(\omega_2 + \omega_1)}$$

(5)

2.1.1 Modeling of agedconcrete

In normal practice, it is assumed that all the cement particles in concrete get hydrated in 28 days and concrete get its full compressive strength. But in reality concrete gains some compressive strength with age beyond 28 days.On the other hand, the durability of concrete is considerably affected due to damage resulting from time variant external loading, moisture and heat transport, freez-thaw actions,chemically expansive reaction and chemical dissolution. Out of the wide range of environmental induced mechanisms, damage due to chemical and mechanical degradation is modeled here to get reasonable elasticity properties of concrete at different ages of concrete.

2.1.2 Gain in compressive strength with time

The gain of compressive strength of concrete is predicted by curve fitting on 50 years of experiential data published by Washa el al. [17]. The authors proposed four different curves for different concretes. All the curves showed an increase in compressive strength roughly proportional to the logarithm of age during the first 10 years and very small variation thereafter.Gogoi and Maity [15] carried out a least square curve fitting analysis on the set of compressive strength data published by Washa el al. [17] and proposed following equation to predict the gain of compressive strength with passage of time in years.

$$f(t) = 3.57\ln(t) + 44.33$$

(6)

where $f(t)$ is the gain of compressive strength in SI unit, t is the age of concrete in years.The value of static modulus of elasticity of concrete in SI unit is obtained by the expression proposed by Neville and Brooks [18].

$$E_0 = 5000\sqrt{f(t)}$$

(7)

2.1.3 Degradation model forconcrete

In present analysis, the degradation of concrete strength is described by thereduction of the net area capable of supporting stresses. The lossof elastic properties of concrete follows as a consequence of degradation of concrete due to various environmental and loading conditions. The orthotropic degradation index is given by

$$d_{gj} = \frac{\Psi_j - \Psi_j^d}{\Psi_j}$$

(8)

where, ψ_j istributary area of the surface in direction j; and ψ_j^d is area affected by degradation. In a scale of 0 to 1, the orthotropic degradation index, d_{gj}=0, indicates no degradation and d_{gj}= 1, indicates completely degraded material. The index j=1,2 corresponds with the Cartesian axes x and y in the two dimensional case. The effective constitutive relationship for plane strain analysis can be expressed as

$$[D_g] = \frac{E_0}{(1+m)(1-2m)} \begin{bmatrix} (1-m)L_1^2 & ML_1L_2 & 0 \\ ML_1L_2 & (1-m)L_2^2 & 0 \\ 0 & 0 & (1-2m)L_1^2L_2^2/(L_1^2+L_2^2) \end{bmatrix} \quad (9)$$

where, $\Lambda_1 = (1 - d_{g1})$ and $\Lambda_2 = (1 - d_{g2})$. In the above equation, E_0 is the elastic modulus of the material without degradation. If $d_{g1} = d_{g2} = d_g$, the isotropic degradation model is expressed as

$$[D_g] = (1 - d_g)^2 [D] \quad (10)$$

where $[D_g]$ and $[D]$ are the constitutive matrices of the degraded and un-degraded model respectively.

2.1.4 Evaluation of degradation index

The compressive strength of concrete is expected to decrease with its age due to chemical and mechanical degradation and this degradation is measured in terms of degradation index. In present work, the degradation due to hygro-chemo-mechanical actions is implemented and the total porosity, ϕ which is the sum of the initial porosity ϕ_0, the porosity due to matrix dissolution ϕ_c and the apparent porosity ϕ_m is considered as the measurement of degradation index. Bangert et al. [19] and Kuhl et al. [14] have suggested the following relationship to relate these parameters.

$$\phi = [\phi_0 + \phi_c + \phi_m] \quad (11)$$

The parameter ϕ_m is obtained as

$$\phi_m = [1 - \phi_0 - \phi_c] d_g \quad (12)$$

Here, d_g is the scalar degradation index. Gogoi and Maity[15] proposed the following equation to obtained degradation index.

$$d_g = \alpha_s - \frac{k^0}{k} \left[1 - \alpha_c + \alpha_c e^{(\beta_c[k^0 - k])} \right] \quad (13)$$

where k^0 and k are values of strain that represent the initial threshold degradation and the internal variable defining the current damage threshold depending on the loading history. and α_s, α_c and β_c are material parameters.

k^0 is given by f_1/E_0, where f_1 is the static tensile strength and E_0 is the elastic modulus of non-degraded material. For no degradation due to mechanical loading, k may be considered equal to k^0. Bangert et al. [19] outlined the procedure to calculate the values of parameter α and β_c. The values of α_s is considered to lie between 1.0 and 0.0, indicating complete and no degradation respectively. Atkin [20] also made a study on the process of degradation and introduced a new parameter ζ. The value of ζ is zero for fresh concrete and $\zeta = 1$ for fully degraded concrete and proposed the following relationship.

$$\dot{\varsigma} = \frac{1}{T_a}\left[1-\varsigma\right] \tag{14}$$

where T_a=design life of the structure. Integrating eq. (14) the following equation can be obtained.

$$1-\varsigma = e^{\left(-\frac{t}{T_a}\right)} \tag{15}$$

Replacing ζ with d_g in eq. (15),the degradation index with time can be obtained as

$$d_g = 1 - e^{\left(-\frac{t}{T_a}\right)} \tag{16}$$

where t is the time corresponding to which degradation index is required. The relation between degraded modulus of elasticity, E_g and modulus of elasticity after strength gain at a particular age, E_0 is given as

$$E_g = (1-d_g)E_0 \tag{17}$$

The dimensionless total porosity is obtained by the following equation

$$E_g = (1-\phi)^{\frac{t}{T_a}} E_0 \tag{18}$$

2.2 Theoretical formulation for reservoir
Assuming fluid to be linearly compressible, inviscidand with small amplitude irrotational motion, the hydrodynamic pressure distribution due external excitation is given as

$$\nabla^2 p(x, y, t) = \frac{1}{c^2} \ddot{p}(x, y, t) \qquad (19)$$

where c is the acoustic wave velocity in the water and ∇^2 is the Laplacian operator in two dimensions. The pressure distribution in the fluid domain is obtained by solving Eq. (19) with the following boundary conditions. A typical geometry of fluid - structure system is shown in Fig. 1.

i) At surfaceI

Considering the effect of surface wave of the fluid, the boundary condition of the free surface is taken as

$$\frac{1}{g} \ddot{p} + \frac{\partial p}{\partial y} = 0 \qquad (20)$$
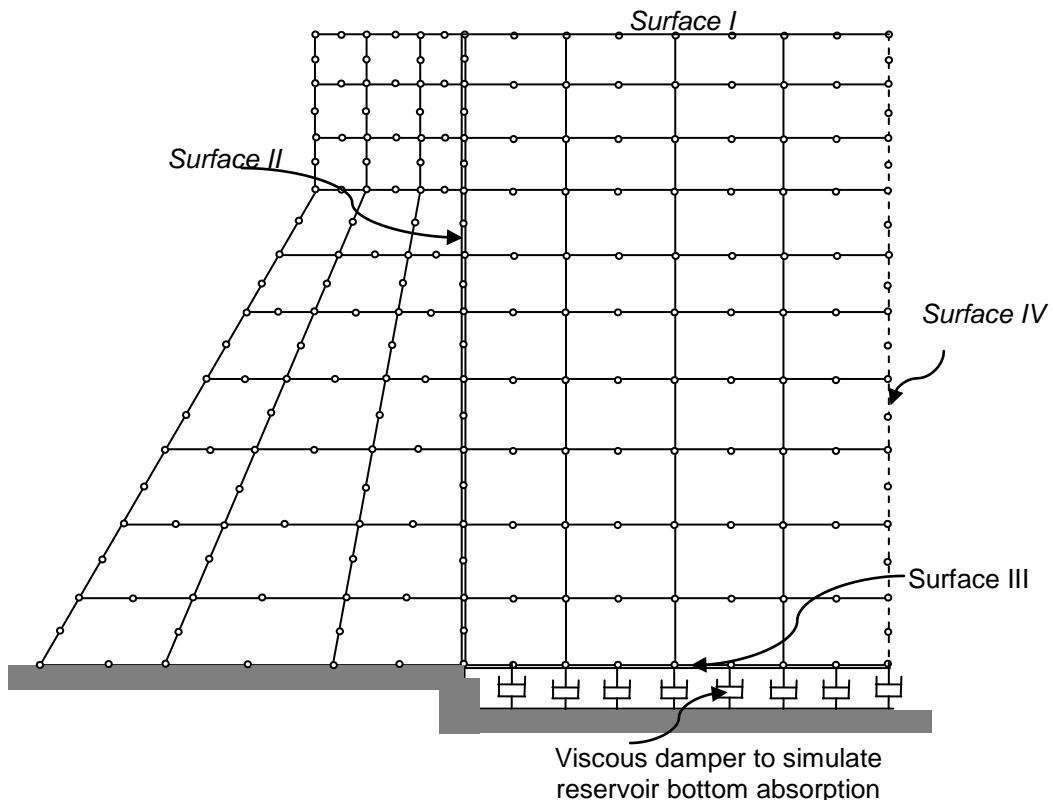
Figure 1. Geometry of dam-reservoir system

ii) At surface II

At fluid-structure interface, the pressure should satisfy

$$\frac{\partial p}{\partial n}(0, y, t) = \rho_f a e^{i\omega t} \qquad (21)$$

where $ae^{i\omega t}$ is the horizontal component of the ground acceleration in which, ω is the circular frequency of vibration and $i = \sqrt{-1}$, n is the outwardly directed normal to the element surface along the interface. ρ_f is the mass density of the fluid.

iii) At surface III

According to the technique proposed by Hall and Chopra [21] reservoir bottom absorption is modeled here. Neglecting the vertical acceleration at reservoir, reservoir bottom absorption may be expressed as:

$$\frac{\partial p}{\partial n}(x,0,t) = -q\dot{p}(x,0,t) \tag{22}$$

Assuming a time harmonic behavior of pressure $p(x,0,t) = p_0(x,0,t)\,e^{i\omega t}$, the eq. (22) may be expressed as:

$$\frac{\partial p}{\partial n}(x,0,t) = i\omega q p(x,0,t) \tag{23}$$

where n is the outwardly directed normal to the element surface and q is a coefficient expressed as:

$$q = \frac{1}{C}\left(\frac{1-\alpha}{1+\alpha}\right) \tag{24}$$

α is the frequency independent reflection coefficient

iv) At surface IV

In case of finite fluid domain, this surface is considered to be rigid and thus the boundary condition in this case will become as follows:

$$\frac{\partial p}{\partial n}(L,y,t) = 0.0 \tag{25}$$

where L is the distance between structural surface and surface IV. In case of infinite fluid domain, the domain needs to be truncated at a suitable distance for the finite element analysis. The truncation boundary as proposed by Gogoi and Maity [22] has been implemented at truncation surface.

The derivation of finite element formulation for the infinite reservoir water is given in Appendix A for ready reference. The finite element form for the reservoir domain is obtained as

$$[E]\{\ddot{P}\}+[A]\{\dot{P}\}+[G]\{P\}=\{F_r\} \tag{26}$$

2.3 Theoretical formulation for dam-reservoir system
In the dam-reservoir interaction problems, the dam and the reservoir do not vibrate as separate systems under external excitations, rather they act together in a coupled way. Therefore, this fluid- structure interaction problem has to be dealt in a coupled way.A direct coupling approach is developed in the present study to obtain the response of dam-reservoir coupled system under external excitation.The coupling of dam and reservoir may be formulated in following way.

The discrete structural equation with damping may be written as:

$$M\ddot{u}+C\dot{u}+Ku=Qp+F_d \tag{27}$$

The coupling term $[Q]$ in eq. (27) arises due to the acceleration and pressure specified onthe dam-reservoir interface boundary and can be expressed as

$$\int_{\Gamma_s} N_u^T npd\Gamma = \left(\int_{\Gamma_s} N_u^T nN_p d\Gamma\right)p = Qp \tag{28}$$

where, n is the direction vector of the normal to the fluid-structure interface. N_u and N_p are the shape functions of dam and reservoir respectively. F_d consists of external forces on dam. Similarly, due to external acceleration from dam body, the fluid equation may bewritten as:

$$E\ddot{p}+A\dot{p}+Gp+Q^T\ddot{u}=F_r \tag{29}$$

Now, the system of Eq. (27) and Eq. (29) are coupled in a second-order ordinary differential equations, which defines the coupled dam-reservoir system completely. These sets of coupledequations are solved on two different meshes of fluid and structure. The eq. (27) and (29)may be written as a set:

$$\begin{bmatrix} M & 0 \\ Q^T & E \end{bmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix}+\begin{bmatrix} C & 0 \\ 0 & A \end{bmatrix}\begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix}+\begin{bmatrix} K & -Q \\ 0 & G \end{bmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix}=-\begin{Bmatrix} F_d \\ F_r \end{Bmatrix} \tag{30}$$

For free vibration analysis, omitting all the damping terms, Eq. (30) can be written as

$$\begin{bmatrix} M & 0 \\ Q^T & E \end{bmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix}+\begin{bmatrix} K & -Q \\ 0 & G \end{bmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix}=0 \tag{31}$$

Natural frequency of dam-reservoir system can be obtained by eigenvalue solution of the Eq. (31). However,the matrices in Eq. (31) are unsymmetrical and standard eigenvalue

solutions can not be used directly. So the above matrices are to be rearranged in a symmetric form. This can be accomplished by change of variables.Now introducing two variables $\tilde{u} = ue^{i\omega t}$ and $\tilde{p} = pe^{i\omega t}$, Eq. (32) can be written as

$$K\tilde{u} - Q\tilde{p} - \omega^2 M\tilde{u} = 0 \tag{32}$$

$$E\tilde{p} - \omega^2 Q^T \tilde{u} - \omega^2 G\tilde{p} = 0 \tag{33}$$

Again introducing another variable $\tilde{q}$ such that

$$\tilde{p} = \omega^2 \tilde{q} \tag{34}$$

After substitution of above three equations in Eq. (31), the final form of this equation becomes

$$\left\{ \begin{bmatrix} K & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & Q \\ 0 & 0 & E \\ Q^T & E^T & G \end{bmatrix} \right\} \begin{Bmatrix} \tilde{u} \\ \tilde{p} \\ \tilde{q} \end{Bmatrix} = 0 \tag{35}$$

Now, the above matrices in dam-reservoir system are symmetric and are in standard form. Further, the variable can now be eliminated by static condensation and the final dam-reservoir system becomes symmetric and still contains only the basic variables.

3. NUMERICAL RESULTS

3.1 Validation of present algorithm

To examine the accuracy of the proposed algorithm, a benchmark problem has been solved and compared with existing literature [24]. The material properties of the coupled system considered, in the present case are same as considered by Samii and Lotfi [24] and given in Table1. Geometric details and a typical finite element discretization for the dam-reservoir system are shown in Fig. 2. The infinite reservoir is truncated at a distance of 200m from the face of the dam and Somerfeld's boundary condition is implemented at truncation surface for the radiating waves as considered by Samii and Lotfi [24]. The first five natural frequencies of the fluid-structure system are listed and compared with those values obtained by Samii and Lotfi [24] in Table 2. In the present analysis, the dam and reservoir domain are discretized by 5 × 10 (i.e., no. of element in horizontal direction, N_h = 5 and number of element in vertical direction, N_v = 10) and 12 ×8 (i.e., N_h = 12 and N_v = 8), respectively. A little variation in the results is observed from Table 2 which may be due to different finite element mesh sizes. However, the comparison of the results shows the accuracy of the developed computer code.

Table 1: Basic parameters of concrete and water

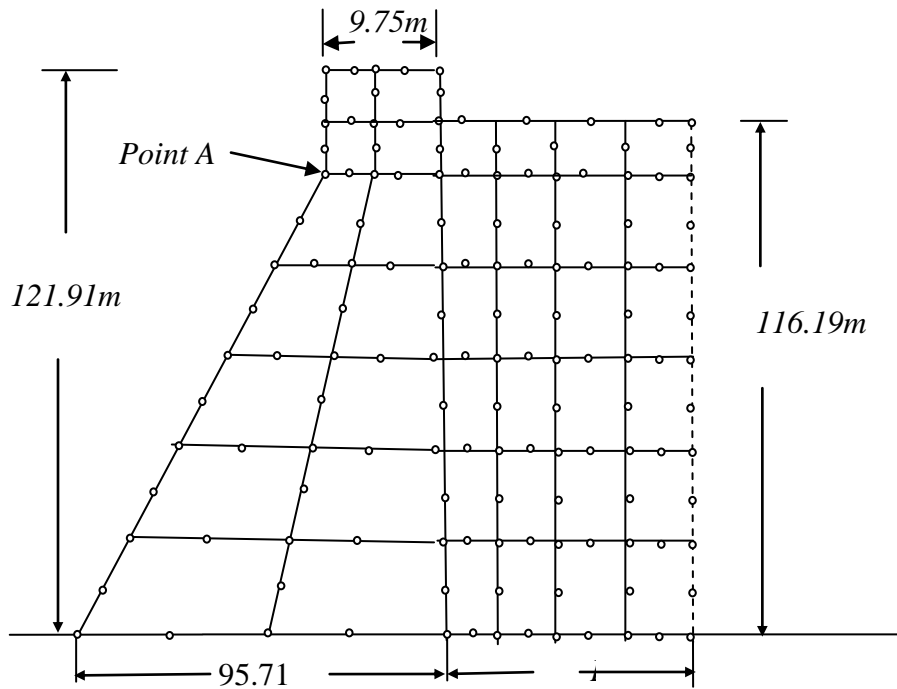| Concrete modulus of elasticity	22.75GPa
Concrete Poisson's ratio	0.2
Unit weight of concrete	2480 kg/m^3
Pressure wave velocity	1440m/s
Unit weight of water	981kg/m^3

Figure 2. Finite element mesh of dam-reservoir system

3.2 Ageing effect on modulus of elasticity of concrete
In normal practice, it is assumed that the concrete get its full compressive strength in 28 days but in reality concrete gains some compressive strength with age beyond 28 days. In present study, this gain of compressive strength is determined from the curve proposed by Gogoi and Maity [15]. The scalar degradation parameter, d_g is evaluated by Eq. (13) using following material properties α_m= 0.9, β_m = 1000, and ϕ_0 = 0.2 (Kuhl et al. [14]). The value of ϕ_c may be considered as 0.2. The allowable degradation due to mechanically induced porosity can be predefined between 1.0 and 0.0, indicating 100 percent and no degradation respectively. Here, the value of a_s for a design life of 100 years is taken in the range of 0.4 to 1.0. The variation of elastic modulus of damaged concrete with design life of 100 years is plotted and compare with those values of without degradation in Fig. 3.
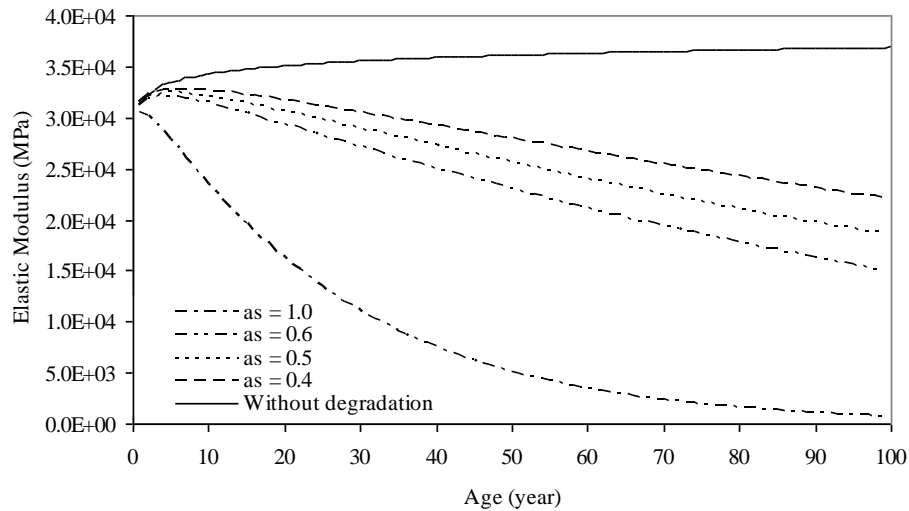
Figure 3. Variation modulus of elasticity with age

It is observed from Fig. 3 that if degradation of concrete is considered, the elastic modulus of the concretedecreases significantly and the decrease is more when the value of α_s is equal to 1.0.

Table 2: The first five natural frequencies of the dam-reservoir system

Mode number	Natural frequency (Hz)	
	Present study	Samii and Lotfi [24]
1	2.5341	2.5267
2	3.2712	3.2681
3	4.5626	4.6651
4	6.2326	6.2126
5	7.9435	7.9181

3.3 Ageing effect on frequency of dam-reservoir coupled system
The ageing effect of concrete on the frequency of dam-reservoir system is studied considering Pine flat dam. For prediction of dynamic behavior of an ageing dam, a new paradigm is introduced defining extent of degradation of the dam due to the effect of hygro-chemo-mechanical (HCM) effects. During the lifetime of the structure, the original safety margin will be reduced by deterioration of structural strength, reflected in the time evolution of the stiffness matrix, the most suitable assemblage of structural degradation information. These degradation of concrete in dam may result in loss of strength of the material along horizontal, vertical or in both the directions. A study is carried out to evaluate the response of the dam due to damage along the width and height of the dam. It is observed from Fig. 4 that with an increase in damage caused by degradation, the natural frequency of the dam reduces. This behavior is mainly due to the reduction stiffnessof the structure with increased

degradation. Moreover, the decrement in frequency is high in case of isotropic damage model as compared to orthotropic degradation. The damage along vertical direction also has a similar effect as that in the isotropic case, as this is the primary source of stiffness.The effect isotropic degradation due tohygro-chemo-mechanical action for different design life structure is also studied. In Table 3 the frequency of Koyna dam of different design life is summarized. The fundamental frequency for a particular design life decreases with the increases of age of the dam. However, the decrease in frequency is comparatively less when the design life of the dam is higher.

Table 3: Fundamental frequency of aged dam

	Fundamental frequency (rad/sec)
No degradation in dam	17.34
After 25 years, design life 50 yrs	11.33
After 50 years, design life 50 yrs	7.98
After 25 years, design life 100 yrs	14.02
After 50 years, design life 100 yrs	11.33

3.4 Response of dam-reservoir coupled system due to harmonic acceleration
In the present section, the ageing effect of concrete in dam is studied considering Pine flat dam which is shown in Fig. 2. The study is carried out with following material and geometric properties: reservoir height =116.19 m;reservoir bottom reflection coefficients as (α) =0.5 and 0.95; age of concrete=immediate after construction, after 25 years and after 50 years; wave velocity = 1440m/s; mass density of water=1000kg/m^3; mass density of concrete=2400kg/m^{3}and design life of the dam=100 years.For finite element implementation, the water is considered as compressible and inviscid and the infinite reservoir is truncated at a distance of 58m from the dam reservoir interface.The truncation boundary condition as proposed by Gogoi and Maity[22] is implemented at truncation surface.Thedam and reservoir are discretized by 6 × 10 (i.e., N_h = 6 and N_v = 10) and 4 ×8 (i.e., N_h = 4 and N_v = 8) respectively. Maximum stresses developed in dam for three different values of excitations(TC/H_f=1,4 and 10) are listed in Table 3.The amplitude of the sinusoidal acceleration is assumed to be 1m/s^2. The no. of time step per cycle of the excitation is taken as 32 after performing convergence study. The maximum stresses developed in the dam is presented in Table 4 correspond to the time 0.75T.In Table 4, it is observed that in case of TC/H_f=1 and TC/H_f=10, the maximum stress at early ages of concrete is larger than that of at later ages. This is mainly due to the larger value of elastic modulus of concrete at early age. On the other hand, for TC/H_f=4 the stress at the age of 50 years is largest compare to the values at the age immediate after construction and 25 years. This happened because the natural frequency of dam-reservoir coupled system for TC/H_f=4 at an age of 50 years is 3.086Hz which is almost equal to the frequency of excitation 3.100Hz.

Development of hydrodynamic pressures on dam are also studied for three different excitation frequencies (TC/H_f=1,4 and 10). Here, the hydrodynamic pressure coefficient is defined as $C_p = P/(a\rho H_f)$. The hydrodynamic pressure coefficient along the upstream face of the dam is plotted in Figs. 5–7 for different ages of dam. Results show that the pressure

coefficient decreases with the increase of age of the dam (i.e., Figs. 5 and 7). However, it is interesting to note that the pressure coefficient increases at the age of 50 years if the excitation frequency is considered as TC/H_f=4. As observed earlier, this happened due to the resonance of coupled system.
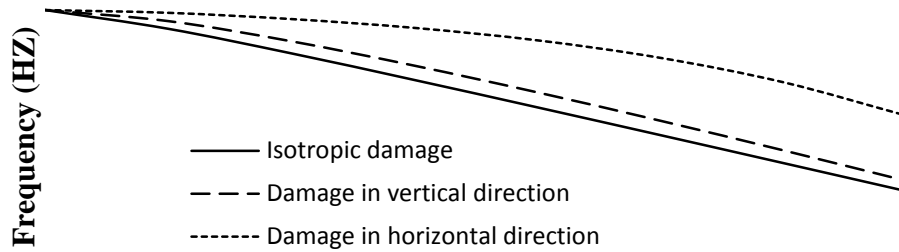
Degradation Index

Figure 4. Variation of frequency of dam with degradation

Table 4: Maximum stresses in concrete dam

| TC/H_f	Reflection coefficient	Age of concrete	Maximum normal stress (N/m^2)
1	0.95	Immediate after construction	3.279×10^5
		25 years	3.252×10^5
		50 years	2.893×10^5
	0.5	Immediate after construction	3.290×10^5
		25 years	3.232×10^5
		50 years	2.877×10^5
4	0.95	Immediate after construction	1.227×10^6
		25 years	1.154×10^6
		50 years	1.352×10^6
	0.5	Immediate after construction	1.236×10^6
		25 years	1.141×10^6
		50 years	1.388×10^6
10	0.95	Immediate after construction	4.723×10^5
		25 years	4.648×10^5
		50 years	3.911×10^5
	0.5	Immediate after construction	4.752×10^5
		25 years	4.677×10^5
		50 years	3.929×10^5
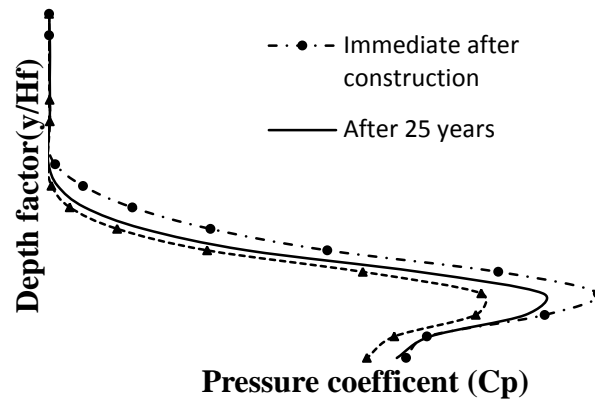
Figure 5. Distribution of hydrodynamic pressure coefficient (C_p) on the face of the dam for $TC/H_f=1$ and $\alpha=0.95$
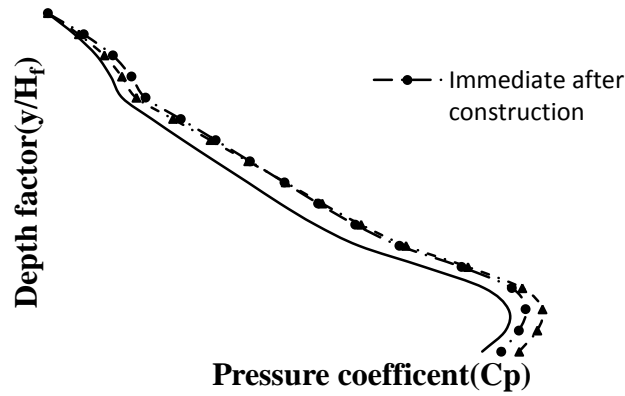

Figure 6. Distribution of hydrodynamic pressure coefficient (C_p) on the face of the dam for $TC/H_f=4$ and $\alpha=0.95$
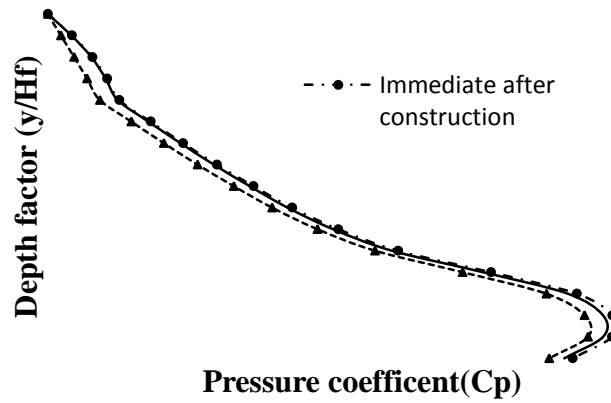

Figure 7. Distribution of hydrodynamic pressure coefficient (C_p) on the face of the dam for $TC/H_f=10$ and $\alpha=0.95$

3.5 Response of dam-reservoir coupled system due to earthquake acceleration

Responses of same pine-flat dam due to earthquake acceleration have also been studiedconsidering reservoir bottom reflection coefficients,α as 0.95 at three different ages of concrete. Material properties and boundary condition at the truncation surface are similar to those consider in the previous case,except the truncation length, which is taken as 350m at which results are converging. The dam and reservoir are discretized by 4×10 (i.e., $N_h = 4$ and $N_v = 10$) and 16×8 (i.e., $N_h = 16$ and $N_v = 8$) respectively. Horizontal component of Koyna earthquake (1967) acceleration is consider as external excitation. The power spectrum of this excitation is shown in Fig. 8. The response of dam-reservoir system is determined in terms of natural frequencies, crest displacements, hydrodynamic pressure coefficients (C_p) at the heel of dam, major and minor principal stresses developedat notch (i.e., point A) of dam. The first three natural frequencies for different ages of dam are given in Table 5. It is evident from Fig. 9 that the crest displacement at the age of 50 years increasesas the modulus of elasticity at this age decreases due to the degradation of concrete. It may be important to note that the magnitude of crest displacement is more at the age of 25 years compare to that at the age of 50 years. This is because the frequency of 25 years aged dam-reservoir system, *i.e.*, 21.34 rad/sec,is close to the dominant frequency of the earthquake acceleration, *i.e.,* 22.06 rad/sec (Fig. 8). In case of developed hydrodynamic pressure (Fig. 10), similar conclusions can be made.

The major and minor principal stresses at notch (i.e., at point A) are also plotted in Figs. 11 and 12 respectively. It is observed that the principal stresses in the dam reduce significantly at the age of 50 years. It is interesting to note that both the principal stresses increase rapidly at the age of 25 years because of occurrence of resonance.
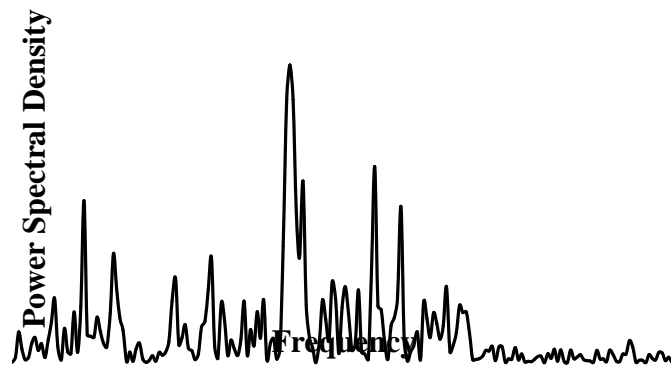

Figure 8. Power Spectrum of earthquake data

Table 5: Natural frequencies of dam-reservoir system with various ages

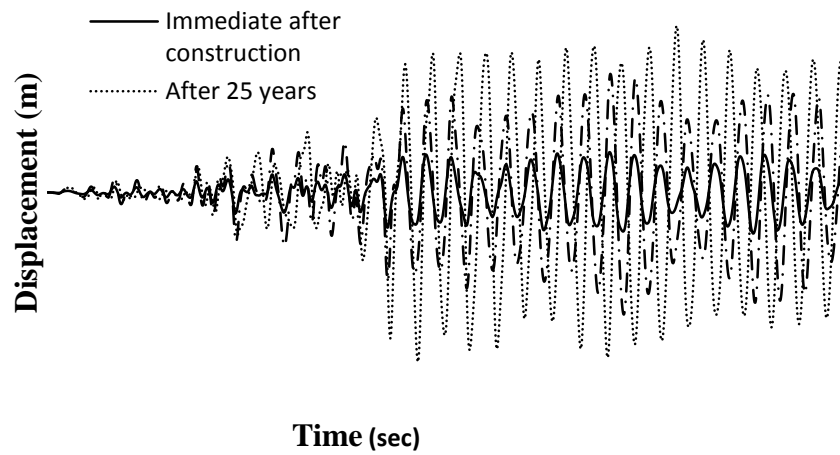| Mode no	Frequency (rad/sec)		
	Immediate after construction	25 Years	50 Years
1	24.01	21.34	19.36
2	29.18	28.51	25.68
3	39.89	34.16	29.89

Figure 9. Horizontal displacement of dam at top due to Koyna earthquake for different ages
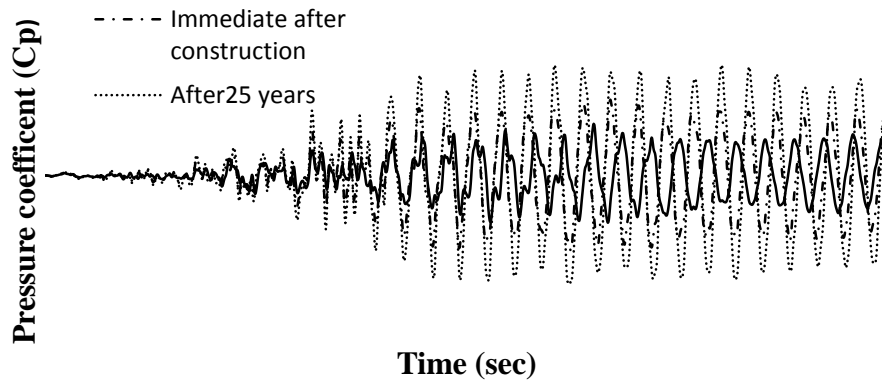
Figure 10. Hydrodynamic pressure at the heel of the dam due to Koyna earthquake for different age
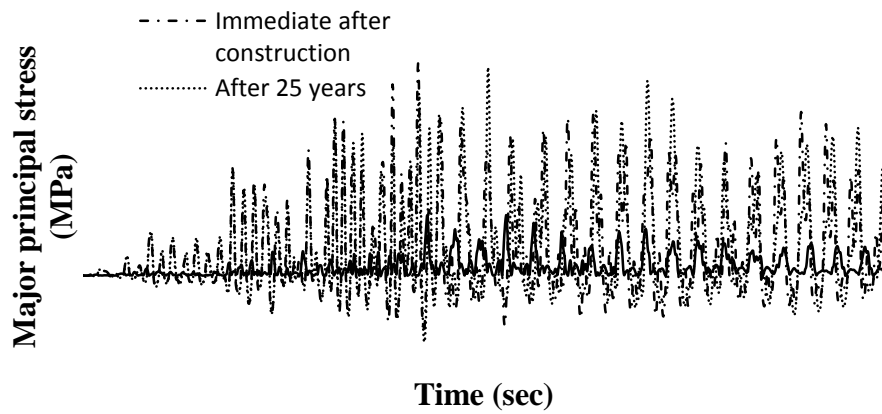
Figure 11. Major principal stress at point A of dam due to Koyna earthquake for different ages
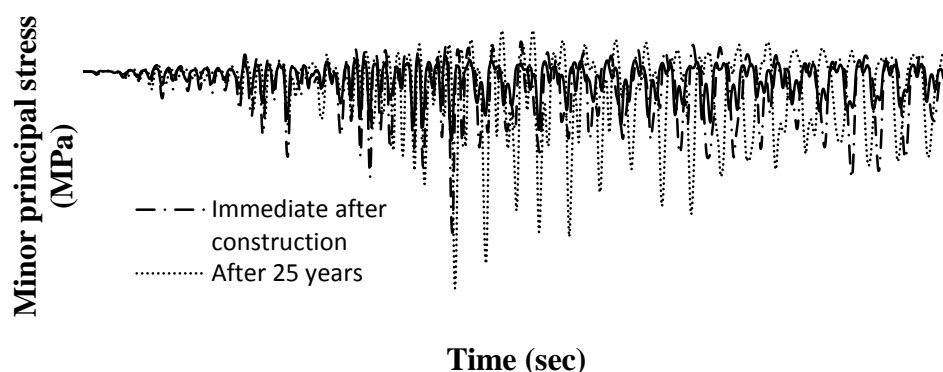
Figure 12. Minor principal stress at point A of dam due to Koyna earthquake for different ages

4. CONCLUSIONS

The response of coupled dam-reservoir system at its different age under external excitation is presented. The hygro-chemo-mechanical degradation causes decreases in modulus of elasticity that leads to the changes in the behavior of dam with time. The dam remains relatively stiff at early age. As a result, the hydrodynamic pressure and stresses on dam have higher values. In general, the stresses in dam decrease with its age due to reduction of modulus of elasticity of concrete because of degradation. However, in the present study it is observed that the magnitude of stresses increases at the age of 50 years under harmonic excitation. Such type of behavior at the particular age is observed because of the excitation frequency which is close to the natural frequency of the aged dam-reservoir coupled system. Similar trend is observed at the age of 25 years due to earthquake excitation. At the age of 25 years, the frequency of coupled system is quite closer to the dominant frequency of the earthquake excitation and as a result, the stresses on dam become unexpectedly larger which may causes failure of dam. Therefore, dam at a particular location should be designed in such a way that the natural frequency of dam-reservoir system at its different ages always remains less than the dominant frequency of previously occurred earthquake at that location.

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APPENDIX A

Finite element implementation for infinite fluid domain

By using Galerkin approach and assuming pressure to be the nodal unknown the discretized form of equation (16) may be written as

$$\int_{\Omega} N_{rj} \left[\nabla^2 \sum N_{ri} p_i - \frac{1}{c^2} \sum N_{ri} \ddot{p}_i \right] d\Omega = 0 \qquad \text{(A-1)}$$

where, N_{rj} is the interpolation function for the reservoir and Ω is the region under consideration. Using Green's theorem equation (A-1) may be transformed to

$$-\int_{\Omega} \left[\frac{\partial N_{rj}}{\partial x} \sum \frac{\partial N_{ri}}{\partial x} p_i + \frac{\partial N_{rj}}{\partial y} \sum \frac{\partial N_{ri}}{\partial y} p_i \right] d\Omega$$

$$-\frac{1}{c^2} \int_{\Omega} N_{rj} \sum N_{ri} \, d\Omega \; \ddot{p}_i + \int_{\Gamma} N_{rj} \sum \frac{\partial N_{rj}}{\partial n} d\Gamma \; p_i = 0 \qquad \text{(A-2)}$$

in which *i* varies from 1 to total number of nodes and Γ represents the boundaries of the fluid domain. The last term of the above equation may be written as

$$\{B\} = \int_{\Gamma} N_{rj} \frac{\partial p}{\partial n} d\Gamma \qquad \text{(A-3)}$$

The whole system of equation (A-3) may be written in a matrix form as

$$\left[\bar{E} \right] \{\ddot{P}\} + \left[\bar{G} \right] \{P\} = \{F\} \qquad \text{(A-4)}$$

where,

$$\left[\bar{E}\right]=\frac{1}{C^2}\sum\int_{\Omega}\left[N_r\right]^T\left[N_r\right]d\Omega \tag{A-5}$$

$$\left[\bar{G}\right]=\sum\int_{\Omega}\left[\frac{\partial}{\partial x}\left[N_r\right]^T\frac{\partial}{\partial x}\left[N_r\right]+\frac{\partial}{\partial y}\left[N_r\right]^T\frac{\partial}{\partial y}\left[N_r\right]\right]d\Omega \tag{A-6}$$

$$\left[F\right]=\sum\int_{\Gamma}\left[N_r\right]^T\frac{\partial p}{\partial n}d\Gamma=\left\{F_f\right\}+\left\{F_{dw}\right\}+\left\{F_{fw}\right\}+\left\{F_t\right\} \tag{A-7}$$

Here the subscript f, dw, fw and t stand for the free surface, dam–water interface, reservoir bottom–water interface and truncation surface respectively. As the surface wave is neglected

$$\left\{F_f\right\}=0 \tag{A-8}$$

At the dam–reservoir interface if $\{a\}$ is the vector of nodal accelerations of generalized coordinates, $\{F_{dw}\}$ may be expressed as

$$\left\{F_{dw}\right\}=-\rho\left[R_{dw}\right]\{a\} \tag{A-9}$$

In which,

$$\left[R_{dw}\right]=\sum\int_{\Gamma_{dw}}\left[N_r\right]^T\left[T\right]\left[N_d\right]d\Gamma \tag{A-10}$$

where T is the transformation matrix at dam water interface and N_d is the shape function of dam. At reservoir bottom-water interface

$$\left\{F_{fw}\right\}=i\omega\left[R_{fw}\right]\{p\} \tag{A-11}$$

where,

$$\left[R_{fw}\right]=\sum\int_{\Gamma_{fw}}\left[N_r\right]^T\left[N_r\right]d\Gamma \tag{A-12}$$

And at the truncation boundary,

$$\left\{F_t\right\}=\varsigma\left[R_t\right]\{p\}-\frac{1}{C}\left[R_t\right]\{\dot{p}\} \tag{A-13}$$

$$\left[R_t\right]=\sum\int_{\Gamma_t}\left[N_r\right]^T\left[N_r\right]d\Gamma \tag{A-14}$$

After substitution all terms the equation (A-4) becomes

$$[E]\{\ddot{P}\} + [A]\{\dot{P}\} + [G]\{P\} = \{F_r\} \tag{A-15}$$

where,

$$[E] = [\bar{E}] \tag{A-16}$$

$$[A] = (\frac{1}{C} - \zeta_m)[R_t] \tag{A-17}$$

$$[G] = [\bar{G}] - i\omega q[R_{fw}] \tag{A-18}$$

$$\{F_r\} = -\rho[R_{dw}]\{a\} \tag{A-19}$$

For any given acceleration at the dam-reservoir interface, eq. (A-15) is solved to obtain the hydrodynamic pressure within the reservoir.